Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-free

ATAR course examination 2016

Marking Key

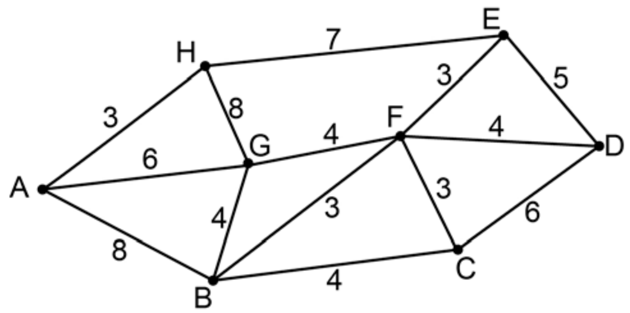
Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2016/64861

Section One: Calculator-free **35% (50 marks)**

Question 1 **(7 marks)**

Joe wishes to upgrade his sprinkler system using the least possible length of piping. The weighted graph below shows the existing system. The numbers on the edges indicate the length of each pipe, in metres, between sprinklers A, B, C, D, E, F, G and H.



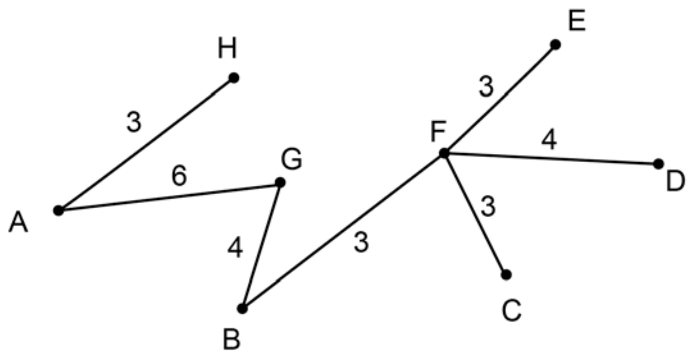
(a) Complete the table below showing connections between each sprinkler. (2 marks)

	A	B	C	D	E	F	G	H
A	-	8					**6**	3
B	8	-	**4**			**3**	**4**	
C		**4**	-	6		**3**		
D			6	-	**5**	**4**		
E				**5**	-	3		7
F		**3**	**3**	**4**	3	-	**4**	
G	**6**	**4**				**4**	-	**8**
H	3				7		**8**	-

Solution
see table above
Specific behaviours
✓ determines approximately 12 or more correct entries ✓ determines all 18 correct entries

(b) Show the use of Prim's algorithm to establish a minimum spanning tree for the least length of piping required and draw this tree on the diagram below. (5 marks)

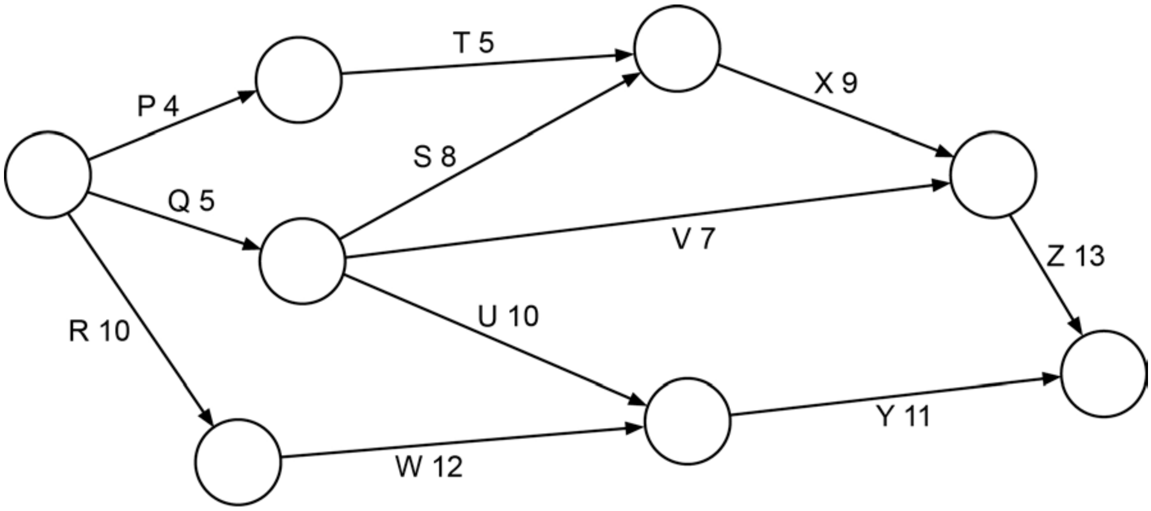
	A	B	C	D	E	F	G	H
A	–	8					6	3
B	8	–	4			3	4	
C		4	–	6		3		
D			6	–	5	4		
E				5	–	3		7
F		3	3	4	3	–	4	
G	6	4				4	–	8
H	3				7		8	–



Solution
see table and graph above (table may have alternative solutions)
Specific behaviours
✓ determines three or more correct links ✓ determines five or more correct links ✓ determines seven correct links ✓ correctly draws tree ✓ attaches correct pipe lengths

Question 2 **(7 marks)**

A project consists of 11 activities, P to Z. The project network representing the scheduling of these activities is shown below. The times are in days.



(a) State the critical path and the minimum completion time for this project. (2 marks)

Solution
critical path QSXZ minimum completion time – 35 days
Specific behaviours
✓ states correct critical path ✓ states correct minimum completion time

(b) Determine the:

(i) earliest starting time for activity Y. (1 mark)

Solution
EST for activity Y is day 22
Specific behaviours
✓ states correct time

(ii) latest starting time for activity V. (1 mark)

Solution
LST for activity V is day 15
Specific behaviours
✓ states correct time

(iii) float time for activity U. (1 mark)

Solution
float time for activity U is nine days
Specific behaviours
✓ states correct time

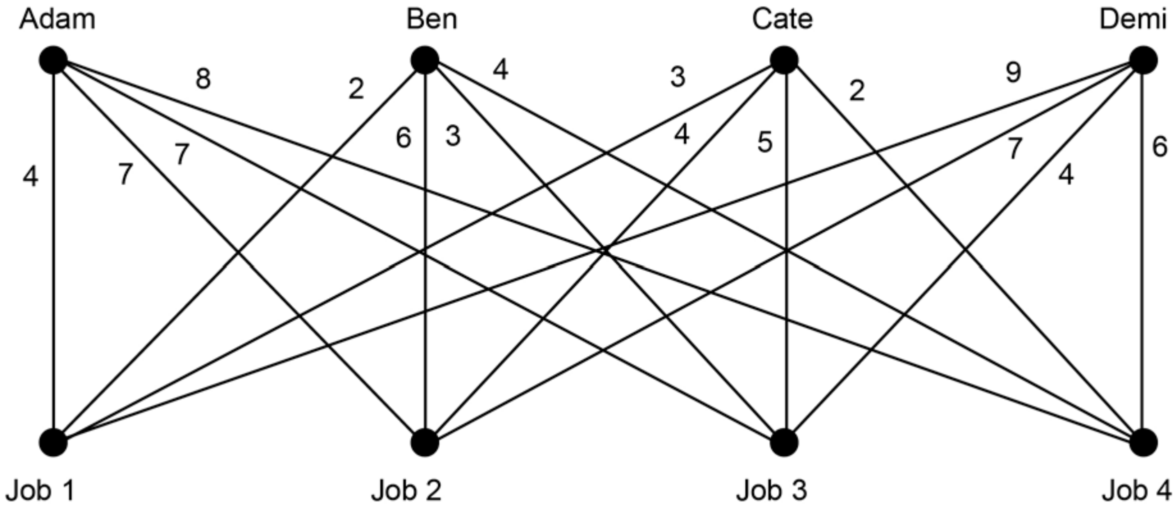
(c) Activity W is delayed by three days. How, if at all, will this affect the critical path and minimum completion time for this project? A copy of the network is given below.

(2 marks)

Solution
critical path changes to RWY minimum completion time is now 36 days (an extra day)
Specific behaviours
✓ states new critical path ✓ states new completion time

Question 3 **(8 marks)**

A foreman in a factory has four workers, Adam, Ben, Cate and Demi, and four jobs to complete. The time, in hours, each worker can complete a particular job is given in the weighted bipartite graph below.



(a) Complete the matrix associated with the bipartite graph above. (2 marks)

$$\begin{array}{c} \\ \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{J1} & \mathbf{J2} & \mathbf{J3} & \mathbf{J4} \end{array} \\ \begin{pmatrix} 4 & \boxed{7} & 7 & \boxed{8} \\ \boxed{2} & 6 & 3 & \boxed{4} \\ 3 & \boxed{4} & \boxed{5} & 2 \\ \boxed{9} & 7 & \boxed{4} & 6 \end{pmatrix} \end{array}$$

Solution
see matrix above
Specific behaviours
✓ completes four or more correct entries ✓ completes eight correct entries

(b) Using the Hungarian algorithm, determine which job the foreman should assign to each of his workers so that the total time is minimised. (6 marks)

Solution

R1 – 4, R2 – 2, R3 – 2, R4 – 4 gives

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} J1 & J2 & J3 & J4 \end{array} \\ \begin{pmatrix} 0 & 3 & 3 & 4 \\ 0 & 4 & 1 & 2 \\ 1 & 2 & 3 & 0 \\ 5 & 3 & 0 & 2 \end{pmatrix} \end{array}$$

C1 – 0, C2 – 2, C3 – 0, C4 – 0 gives

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} J1 & J2 & J3 & J4 \end{array} \\ \begin{pmatrix} 0 & 1 & 3 & 4 \\ 0 & 2 & 1 & 2 \\ 1 & 0 & 3 & 0 \\ 5 & 1 & 0 & 2 \end{pmatrix} \end{array} \sim \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} J1 & J2 & J3 & J4 \end{array} \\ \begin{pmatrix} \cancel{0} & 1 & 3 & 4 \\ \cancel{0} & 2 & 1 & 2 \\ \cancel{1} & \cancel{0} & \cancel{3} & \cancel{0} \\ \cancel{5} & \cancel{1} & \cancel{0} & \cancel{2} \end{pmatrix} \end{array}$$

Number of lines is not equal to the order of the matrix.
Now subtract 1 from all numbers not struck through and add 1 to numbers at the intersection of the strike through. This gives

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} J1 & J2 & J3 & J4 \end{array} \\ \begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 6 & 1 & 0 & 2 \end{pmatrix} \end{array} \sim \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} J1 & J2 & J3 & J4 \end{array} \\ \begin{pmatrix} \cancel{0} & \cancel{0} & \cancel{2} & \cancel{3} \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{1} \\ \cancel{2} & \cancel{0} & \cancel{3} & \cancel{0} \\ \cancel{6} & \cancel{1} & \cancel{0} & \cancel{2} \end{pmatrix} \end{array}$$

Number of lines is now equal to the order of matrix.
Solution is:
Job 1 goes to Ben, Job 2 goes to Adam, Job 3 goes to Demi, Job 4 goes to Cate

Specific behaviours

✓ subtracts lowest values from rows correctly
✓ subtracts lowest values from columns correctly
✓ identifies the number of lines is not equal to order of matrix
✓ subtracts 1 from all numbers not struck through and adds 1 to intersections
✓ identifies the number of lines is now equal to order of matrix
✓ correctly assigns jobs to correct workers

Question 4 **(12 marks)**

(a) Given the sequence 256, 128, 64, 32, …

(i) Write a recursive rule for the sequence. (2 marks)

Solution
$T_{n+1} = \frac{1}{2}T_n\,,\ T_1 = 256$
Specific behaviours
✓ correctly states recursive rule ✓ correctly states first term

(ii) Deduce a rule for the n^{th} term of this sequence. Hence, calculate the 15th term, leaving your answer as a fraction. (3 marks)

Solution
$T_n = 256\left(\frac{1}{2}\right)^{n-1}$ $T_{15} = 256\left(\frac{1}{2}\right)^{15-1} = \frac{1}{64}$
Specific behaviours
✓ correctly states general rule ✓ correctly substitutes 15 into general rule ✓ correctly calculates 15^{th} term as a fraction

(b) Use the recursive definitions given to state the first **three** terms of each of the following sequences.

(i) $T_{n+1} = T_n + 7,\quad T_1 = 11$ (2 marks)

Solution
11, 18, 25
Specific behaviours
✓ correctly calculates term 2 ✓ correctly calculates term 3

(ii) $T_{n+1} = 1.5T_n,\quad T_2 = 7.5$ (2 marks)

Solution
$T_1 = 7.5 \div 1.5 = 5$ $T_3 = 7.5 \times 1.5 = 11.25$
Specific behaviours
✓ correctly calculates term 1 ✓ correctly calculates term 3

(c) Consider the sequence 12, 7, 2, –3, …

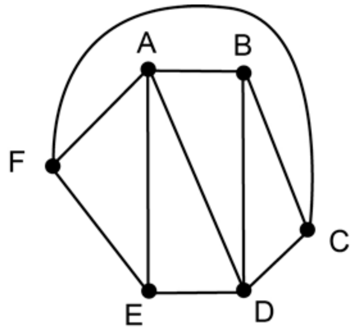
By deducing a rule for the n^{th} term, or otherwise, determine which term of the sequence is –168. (3 marks)

Solution
$T_n = 12 + (-5)(n-1)$ $17 - 5n = -168$ $5n = 185 \Rightarrow n = 37$
Specific behaviours
✓ correctly states general term ✓ correctly equates to –168 ✓ correctly states correct term

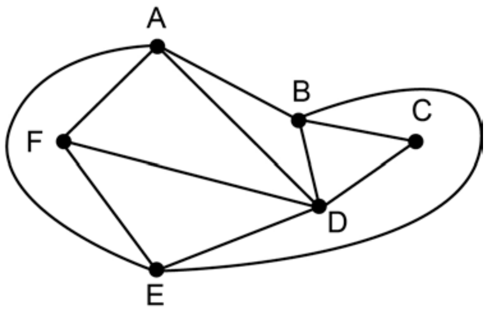
Question 5 **(9 marks)**

(a) Redraw the following graphs as planar graphs.

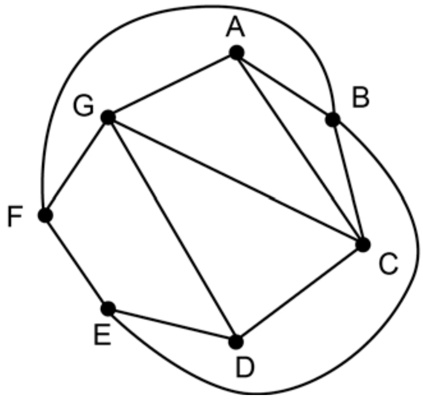
(i) (1 mark)

Solution

Specific behaviours
✓ correctly draws planar graph

(ii) (2 marks)

Solution

Specific behaviours
✓ correctly redraws edge AE ✓ correctly redraws edge BE

(iii) (2 marks)

Solution

Specific behaviours
✓ correctly redraws edge FB ✓ correctly redraws edge BE

(b) Verify Euler's formula for the planar graph obtained in part (a)(i). (2 marks)

Solution
$v = 6, f = 6, e = 10$ $v + f - e = 6 + 6 - 10 = 2$ which verifies Euler's formula
Specific behaviours
✓ correctly determines number of vertices, faces and edges ✓ verifies Euler's formula

(c) One of the planar graphs is semi-Eulerian. State which graph it is, giving a reason for your choice. (2 marks)

Solution
graph (ii) is semi-Eulerian since it has exactly two odd vertices
Specific behaviours
✓ identifies correct graph ✓ states correct reason

Question 6 **(7 marks)**

Before a fitness campaign at a high school started, 50 students were chosen at random from each year group and asked the following questions:

Question 1: Which one of the following modes of transport do you use to travel to and from school?

Category A: walking/cycling
Category B: public transport
Category C: private car

Question 2: Which year group are you in?

The campaign organisers wished to determine whether age group affected the students' likelihood of walking/cycling to and from school.

The results of the survey are shown in the table below.

	Category A	Category B	Category C	Total
Year 7	19	**11**	20	50
Year 8	12	17	21	50
Year 9	13	14	23	50
Year 10	11	**18**	21	**50**
Year 11	10	15	25	50
Year 12	8	17	25	50
Total	73	**92**	135	300

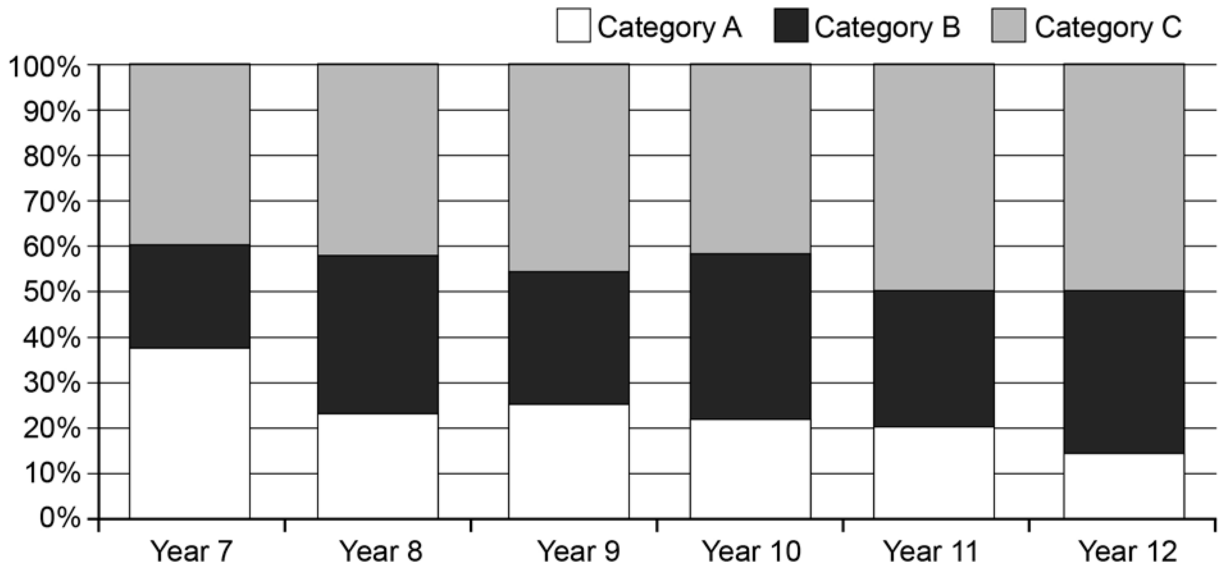
(a) Complete the missing entries in the table above. (2 marks)

Solution
see table above
Specific behaviours
✓ calculates two or more correct entries ✓ calculates four correct entries

(b) Compare the percentages of students in Year 7 and Year 12 who use Category A as a mode of transport and comment on your results. (2 marks)

Solution
Year 7: $\frac{19}{50} = 38\%$, Year 12 : $\frac{8}{50} = 16\%$ There is a marked drop between Year 7 and 12 in the percentage of students who use Category A as a mode of transport to and from school
Specific behaviours
✓ calculates the correct percentages ✓ states there is a drop between Year 7 and Year 12 students using Category A

The data given in the table for part (a) have been displayed as a divided column graph below.



(c) Using the graph above or another method, comment on:

(i) the association between 'Year group' and 'Category A'. (1 mark)

Solution
Generally as the students get older the percentage of students using Category A as a mode of transport decreases.
Specific behaviours
✓ states correct association between year group and category A

(ii) the association between 'Year group' and 'Category C'. (1 mark)

Solution
Generally as the students get older the percentage of students using Category C as a mode of transport increases.
Specific behaviours
✓ states correct association between year group and category C

(iii) the association between 'Category A' and 'Category B and C combined'. (1 mark)

Solution
There are less students who walk or cycle to and from school than those who use motorised transport. **or** As category A increases, Category B and C decrease.
Specific behaviours
✓ states correct association between Category A and Category B and C combined

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